Prepayment option and the interest rate differential between a fixed- and floating-rate mortgage loan

Abstract

Previous studies identify the interest rate differential between a fixed- and floating-rate mortgage loan as the most important factor in explaining mortgage choices. We argue that this differential is a premium paid by a fixed-rate borrower, who may exercise the option to pay off the loan on each payment date after the payment of certain penalties. The borrower will prepay the balance of the loan and apply for a new loan if the market interest rate falls sufficiently below the contract rate. Taking this option value into account, we derive an equilibrium condition that determines the interest rate differential. We find that the differential expands when the term of the loan lasts longer or when interest rates increase at an unpredictable rate because the fixed-rate borrower will then have a more valuable option to prepay the loan.

Keywords: interest rate differential, mean-reverting process, mortgage loans, prepayment option

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I. Introduction

In most countries, lenders provide both fixed- and floating-rate mortgage loans to borrowers. Fixed-rate mortgage loans (FRMs) flourished after the Great Depression in the 1930s. Floating-rate mortgage loans, also known as the adjustable mortgage loans (ARMs) in the U.S., began to appear in 1981. Since then, there have been numerous theoretical and empirical studies investigating the determinants of mortgage choices (see, e.g., Follain, 1990; Koijen, Hemert and Nieuwerburgh, 2009). Empirical studies typically find that the interest rate differential is the most important factor in determining mortgage choices, while household characteristics play a minor role.[1]

The standard literature on mortgage choices treats the interest rate differential as exogenously given. However, this differential should make an investor indifferent

[1] The theoretical literature searches for household characteristics that affect mortgage choices. For example, Baesel and Biger (1980) show that, when the correlation between the rate of inflation and borrowers' labor income is low, a borrower will prefer FRMs to ARMs. Statman (1982) notes that mortgage choices depend on the expected appreciation of housing prices, while Aim and Follain (1984) indicate that mortgage choices depend on maximum payment-to-income ratios and positive net worth constraints. More recently, Campbell and Cocco (2003) argue that "*households with smaller houses relative to income, more stable income, lower risk aversion, more lenient treatment in bankruptcy and higher probability of moving should be the households that find ARMs more attractive*." There are only several empirical studies that investigate the determinants of mortgage choices. Dhillon et al. (1987) use a sample of approximately 80 borrowers from the Baton Rouge office of a national U.S. mortgage bank between January 1983 and February 1984 and find that borrower characteristics have a very weak effect on mortgage choices, but the interest rate differential significantly matters. Brueckner and Follain (1988) find that borrowers prefer ARMs to FRMs when market interest rates are higher, when they have higher income and higher savings, and when they are likely to move out. Finally, Koijen et al. (2009) find that the bond risk premium is the best proxy for the interest rate differential that determines mortgage choices.

between a fixed- or floating-rate mortgage loan offered by the lender and is thus endogenously determined. We thus build a simplified option pricing model in which all mortgage loans are of the "interest-only" type. The mortgage loan provider usually charges a higher interest rate for a fixed-rate loan because the borrower preserves the option to refinance the loan. To exercise this option, the fixed-rate borrower must pay certain penalties, which are typically proportional to the loan balance. The fixed-rate borrower will pay off and refinance the loan only if these penalties are exceeded by the interest payments that would have otherwise been paid over the term of the loan. In other words, a fixed-rate borrower will refinance a loan only if the market interest rate falls sufficiently below its contract rate. Assuming that market interest rates follow a mean-reverting process (Cox, Ingersoll and Ross, 1985), we value the prepayment option owned by the fixed-rate borrower and derive an equilibrium condition that determines the interest rate differential between a fixed- and floating-rate mortgage loan.

By employing plausible parameter values, our numerical analysis indicates that the interest rate differential expands when (i) the growth rate of interest rates fluctuates more severely or (ii) the term of the loan lasts longer, as both scenarios provide a fixed-rate borrower with more opportunities to pay off the loan balance prior to the maturity date. The latter result is consistent with the liquidity preference hypothesis (Hicks, 1946), which predicts that the term premium is higher for bonds with a longer maturity. This result contrasts with other theories, such as the market segmentation hypothesis (Culbertson, 1957) and the preferred habitat theory (Modigliani and Sutch, 1966), neither of which restrict term premiums as monotonically increasing with the term of the loan.

Our paper is related to Schwartz and Torous (1992) and Hilliard, Kau and Slawson (1998), both of which allow for interactions between prepayment and default options.

However, the former assumes that the effect of mortgage age on prepayment behavior is an exogenously given hazard function, while the latter defines the value of immediate prepayment as the value of the remaining payments net of the unpaid mortgage balance. Both theories also assume that a fixed-rate borrower will not refinance after paying off the loan balance, thus abstracting from the prepayment option on which we focus.[2]

The remaining sections are organized as follows. Section II offers an option pricing model to calculate the value of a fixed-rate mortgage loan that can be refinanced over time. We follow the trinomial-tree lattice model of Hull and White (1990) and calculate the value of the option embedded in the fixed-rate mortgage loan. Section III employs plausible parameter values to investigate the comparative-statics results. Section IV concludes and offers directions for future research.

II. The Model

We build a model that captures the option value of prepayment embedded in a fixed-rate mortgage loan. We focus on an interest-only mortgage loan and introduce two parameters, d and α, where d is the rate of mark-up charges to the fixed-rate borrower, and α denotes the transaction cost on prepayment. The analysis becomes extremely complicated if we consider a constant payment mortgage loan, which is more common than an interest-only loan.

Suppose that we start from $t = 0$. Consider a borrower who applies for a fixed-rate mortgage loan with a principal normalized at one unit, and a contract rate equal to $(1+d)r(0)$ per period, where $r(0)$ denotes the market interest rate at time $t = 0$. We assume that the mortgage matures T periods later, and thus, the borrower

[2] We, however, do not consider the option to default that is possibly owned by both fixed- and floating-rate loan borrowers. Our model thus especially applies to those countries, such as the U.K., Canada, Australia, New Zealand and Ireland, in which mortgage loans are recourse, and therefore, loan borrowers are unable to derive any profits from the default.

will continuously take advantage of the option to refinance the loan until the maturity date. To facilitate the analysis, we will consider the fixed-rate mortgage loan with and without the prepayment option. Without the prepayment option, the value of a fixed-rate mortgage loan at time $t = 0$, denoted by $V(0, r(0), d)$, is given by:

$$V(0,r(0),d) = \sum_{\tau=1}^{T} \frac{(1+d)r(0)}{\prod_{k=1}^{\tau}(1+r(k))} + \frac{1}{\prod_{k=1}^{T}(1+r(k))}, \tag{1}$$

where $\prod_{k=1}^{\tau}(1+r(k))$ denotes a sequence of multiplication from $(1+r(1))$ to $(1+r(\tau))$.

While a fixed-rate borrower typically has the option to prepay, the borrower will incur certain transaction costs in the exercise of this option, which usually have the following two features.[3] First, the transaction costs are proportional to the amount of loan balance (Azevedo-Pereira, Newton and Paxson, 2002). Second, the transaction costs decrease as the maturity date approaches (Daglish and Patel, 2012). Our treatment of transaction costs captures the first feature because we assume that the transaction cost on the refinancing date t is equal to α, which is proportional to the loan balance, 1.

Consider a fixed-rate borrower who has the prepayment option, and denote the value of this loan as $V^f(t, r(t), d)$, where $0 \le t \le T$. During the loan period (0, T), the fixed-rate borrower will pay off and refinance the loan if the cost of prepayment at the next payment date falls sufficiently below the current contract rate; otherwise, the borrower will still be charged at the current contract rate. During the loan period, both fixed- and floating-loan borrowers may choose between a fixed- or floating-rate loan at

[3] These two features are especially apt for the U.K. For Canada and Ireland, the prepayment penalty is based on the income lost if the bank relent the money prepaid for a fixed term equal to the remaining fixed term of the original mortgage but at interest rates prevailing at the time of the mortgage being broken (see, Daglish and Patel, 2012).

the next payment date; while the fixed-rate borrower must pay certain penalties, the floating-rate borrower does not. We will assume that each borrower will always choose the floating-rate loan after paying off the prior loan balance. This seems to be innocuous to our conclusion, while substantially simplifying our analysis. As such, we can write the value of the floating-rate mortgage loan at time t, denoted by $V^a(t, r(t))$, as

$$V^a(t, r(t)) = \sum_{\tau=t+1}^{T} \frac{r(\tau)}{\prod_{k=t+1}^{\tau}(1+r(k))} + \frac{1}{\prod_{k=t+1}^{T}(1+r(k))}, \tag{2}$$

The amount of the prepayment cost is pivotal for a fixed-rate borrower in determining whether he/she should refinance the loan. During the loan period, $0 < t < T$, the fixed-rate borrower will (will not) refinance the loan if

$$V^a(t, r(t)) + \alpha < (>)\ V^f(t, r(t), d). \tag{3}$$

The intuition behind Equation (3) is as follows. The fixed-rate borrower can pay α to replace the existing debt obligation $V^f(t, r(t), d)$ by $V^a(t, r(t))$ and will thus do (not do) so if the total cost, $V^a(t, r(t)) + \alpha$, falls short of (exceeds) the benefit of extinguishing the existing debt obligation, $V^f(t, r(t), d)$. Given that a fixed-rate borrower is unaffected by any interest rate increases from the previous payment date, he/she will refinance only if the current market interest rate falls below the market interest rate on the previous payment date.

At the initial date $t = 0$, in equilibrium, a borrower should be indifferent between applying for a floating-rate mortgage loan, whose value is given by $V^a(0, r(0))$, as shown in Equation (2), and a fixed-rate mortgage loan that may be refinanced over time, $V^f(0, r(0), d)$. Note that the fixed-rate borrower is charged at a higher initial contract rate and incurs certain transaction costs from refinancing but enjoys the option to refinance at each payment date. The risk premium paid by the fixed-rate borrower, i.e., the interest rate differential in equilibrium, denoted by d^*, is thus the d that satisfies the equality

$$V^f(0, r(0), d) = V^a(0, r(0)). \tag{4}$$

To value both the fixed- and floating-rate mortgage loan mentioned above, we assume that the market interest rate at instant t, $r(t)$, follows the mean-reverting process, as in Cox et al. (1985):

$$dr(t) = a(b - r(t))dt + c\sqrt{r(t)}dZ(t),\ a > 0,\ b > 0,\ c > 0. \tag{5}$$

The term $Z(t)$ represents a Wiener process, and all a, b and c represent the parameters. The equation states that, on average, the interest rate converges toward b. The rate of convergence is governed by the value of a, and the volatility of the growth rate of interest rates, i.e., $dr(t)/r(t)$, is given by $c\big/\sqrt{r(t)}$.[4] Note that Equation (5) lacks an interest-rate risk premium because we assume that such premium has been absorbed by parameters a and b (Cox, Ross and Rubenstein, 1979).

Suppose that $U(t, r(t), d)$ denotes the value of either a floating- or fixed-rate mortgage loan. Given the evolution of $r(t)$ shown by Equation (5), it follows that $U(\cdot)$ satisfies the partial differential equation given by

$$\frac{c^2}{2}r(t)\frac{\partial^2 U(\cdot)}{\partial r(t)^2} + a(b - r(t))\frac{\partial U(\cdot)}{\partial r(t)} + \frac{\partial U(\cdot)}{\partial t} - r(t)U(\cdot) + S(t, r(t), d) = 0, \tag{9}$$

where $S(t, r(t), d)$ denotes the payment on date t. In the Appendix, we follow Hull and White (1990) to value both $V^a(0, r(0))$, the value of the floating-rate mortgage

[4] Given that interest rates cannot be negative, we thus focus on the volatility of the growth rate of interest rates rather than the volatility of interest rates.

loan defined in Equation (2), and $V^f(0, r(0), d)$, the fixed-rate mortgage loan with prepayment options. Equating $V^a(0, r(0))$ with $V^f(0, r(0), d)$ then yields the interest rate differential in equilibrium, d^*.

III. Numerical Examples

To investigate the determinants of the interest rate differential, we choose a set of plausible parameter values that resemble those chosen by Hull and White (1990) and Hilliard, Kau and Slawson (1998): (i) the current interest rate is equal to 6% per year, i.e., $r(t_0) = 0.06$; (ii) the reversion speed of interest rates per year is equal to 20%, i.e., $a = 0.2$; (iii) the long-run steady state of interest rates is equal to 6% per year, i.e., $b = 0.06$; (iv) the parameter c is equal to 0.1 such that the volatility of interest rates evaluated at the current interest rate is equal to $c/\sqrt{r(t_0)} = 24.5\%$ per year; (v) the term of fixed- and floating-rate loans are equal to five years, i.e., $T = 5$; and (vi) the transaction cost parameter $\alpha = 0.2$.[5] Stanton (1995) argues that the explicit monetary costs associated with refinancing typically do not exceed approximately 7% of the remaining principal balance. However, the total costs associated with refinancing are concentrated at approximately 30% to 50% of the remaining principal loan balance. Following Stanton's argument, we choose α such that the total transaction cost on prepayment is equal to 20% of the remaining principal loan balance. Given this set of benchmark parameter values, in equilibrium, the interest rate differential, as denoted by the mark-up ratio d^*, is equal to 9.07%. Because a floating-rate borrower is charged at 6% per year, a fixed-rate borrower is thus charged at 6.54% per year (6% · 1.0907).

We show our simulation results in both Figure 1 and Table 1, which enable us to

[5] The benchmark parameters fit into those countries with high-yield currencies, such as Australia, Canada and New Zealand before the *Global Financial Crisis* starting from 2007.

form several testing hypotheses. Figure 1 presents the results of d^* for changes in loan term T over the region $(1, 30)$, while holding the other parameters at their benchmark values. Figure 1 shows that the interest rate differential is higher for loans with longer maturity periods. The interest rate differential for a 1-year fixed-rate mortgage loan is equal to 1.65%, and for a 30-year fixed-rate mortgage loan increases to 110%. Given that the initial contract rate for a floating-rate loan is equal to 6% per year, the initial contract rate per year for a 1-year fixed-rate loan is thus equal to 6.01%, and the rate for a 30-year fixed-rate loan is equal to 12.6%.

The positive slope shown in Figure 1 reflects the fact that the fixed-rate borrower has more opportunities to pay off the loan balance with a longer term loan. Our results thus indicate that term premiums tend to be higher for mortgage loans with a longer maturity, which is consistent with the liquidity preference hypothesis (Hicks, 1946). Our result, however, contrasts with the predictions of the market segmentation hypothesis (Culbertson, 1957) and the preferred habitat theory (Modigliani and Sutch, 1966), as neither theory restricts the term structure of interest rates.

Insert Figure 1 here

We have two caveats about our hypothesis regarding the term structure of mortgage rates. First, our model considers a “pure” floating-rate mortgage loan without any embedded options. However, as shown by Brueggeman and Fisher (2006), most ARMs in the U.S. have various embedded options, which will thus affect the interest rate differential. Second, our model does not incorporate household characteristics into account. For example, a borrower may be willing to pay a higher premium to a fixed-rate mortgage loan with a longer maturity if the borrower expects that his/her income will grow over time and that he/she will be more likely to pay off principal on the maturity date. This reinforces the effect stated in our result, which predicts that the term structure of mortgage rates will slope upward.

Table 1 presents the results of d^* for changes in reversion speed (per year) a over the region (18%, 22%), the long-run steady state interest rate (per year) b over the region (5%, 7%), the standard deviation (per year) c over the region (8%, 12%), the initial contract rate for the floating-rate loan (per year) $r(t_0)$ over the region (4%, 8%), and the transaction cost parameter α over the region (15%, 25%) while holding the other parameters at their benchmark values. The interest rate differential contracts (d^* decreases) when either (i) the reversion speed increases, as shown in Rows 1 and 2, or (ii) the long-run steady state interest rate increases, as shown in Rows 3 and 4. Conceivably, this finding reflects the fact that the fixed-rate borrower has fewer opportunities to prepay the loan under each scenario.

Rows 5 and 6 in Table 1 show that the interest rate differential expands (d^* decreases) when interest rates become more volatile (c increases). When the volatility of interest rates is equal to 19.6% per year ($c = 8\%$), the differential is equal to 5.5%. When that volatility increases to 29.4% per year ($c = 12\%$), the differential then increases to 12.6%. This result occurs because a larger c implies that interest rates are equally likely to rise or fall by a larger magnitude, thus providing more opportunities for the fixed-rate borrower to prepay the loan balance.

Rows 9 and 10 in Table 1 show that the interest rate differential decreases when the initial interest rate $r(t_0)$ increases. Conceivably, this reflects the fact that a low initial interest rate will increase the volatility of interest rates given by $c/\sqrt{r(t_0)}$ such that the fixed-rate borrower is willing to pay a higher premium.

Finally, Rows 11 and 12 in Table 1 show that the fixed-rate borrower will seek a lower contract rate when the transaction cost of prepayment (α) increases. When the transaction cost of prepayment is equal to 15% of the loan balance ($\alpha = 0.15$), the interest rate differential is equal to 9.73%. When the transaction cost increases to 25% ($\alpha = 0.25$), the differential is equal to 9.04%. This pattern occurs because a higher transaction cost implies that a fixed-rate borrower's opportunity cost of refinancing is higher such that the borrower is only willing to pay lower premiums.

Insert Table 1 here

IV. Conclusion

Previous studies have identified the interest rate differential between a fixed- and floating-rate mortgage loan as the most important factor in explaining mortgage choices. We use an option valuation framework and show that the interest rate differential is endogenously determined. We predict that this differential will expand when the mortgage loan is long-lived, when interest rates become more volatile and when the fixed-rate borrower incurs smaller penalties for prepayment.

The above predictions, however, are based on very simplified assumptions, which may be relaxed as follows. First, we may allow for stochastic movements of housing prices (see, e.g., Ambrose and Buttimer, 2000; Hilliard, Kau and Slawson, 1998; Kau, Keenan, Muller and Epperson, 1994; Leung and Sirmans, 1990) and for stochastic movements of spot interest rates. We can then simultaneously value the option to refinance and the option to default. Second, we may include other factors, such as the loan-to-value ratio, tax consequences, or other types of transaction costs (see, e.g., Kau, Keenan and Kim, 1993; Stanton, 1995) into our framework and then examine whether our results still hold. We leave these extensions for future research.

Appendix: Valuation of the floating-rate mortgage loan, $V^a(t,r(t))$ and the fixed-rate mortgage loan, $V^f(t,r(t),d)$

We first value $V^a(t,r(t))$. Let $\phi(t)=\sqrt{r(t)}$ and define $(\frac{c}{2})^2\Delta t/(\Delta\phi)^2=\frac{1}{3}$, where Δt is a small change in the time interval, and $\Delta\phi$ is a small change in $\phi(t)$. Hull and White (1990) then show that:

$$\phi_{\min}\le\phi(t)\le\phi_{\max}, \qquad \text{where } \phi_{\min}=\left(-\beta+\sqrt{\beta^2+4\alpha_1\alpha_2}\right)/2\alpha_2,$$

$$\phi_{\max}=\left(\beta+\sqrt{\beta^2+4\alpha_1\alpha_2}\right)/2\alpha_2,\ \beta=\frac{\Delta\phi}{2\Delta t},\ \alpha_1=\frac{4ab-c^2}{8}, \text{ and } \alpha_2=\frac{a}{2}.$$

The values of $\phi(t)$ considered on the grid for the explicit finite difference method are ϕ_0, ϕ_1, ..., ϕ_j, …, ϕ_n, where ϕ_0 is the largest multiple of $\Delta\phi$ less than $\phi_{\min}$, $\phi_j=\phi_0+j\Delta\phi$, and n is the smallest integer such that $\phi_n\ge\phi_{\max}$. We also choose $\Delta\phi$ such that ϕ_0 plus some multiple of $\Delta\phi$ equals the current value of $\phi(t_0)$. Similarly, the values of t considered in the grid are t_0, t_1, ..., t_i, …, T, where $t_i=t_0+i\Delta t$. We use the modified explicit finite difference method (Hull and White, 1990) to value $V^a(t,\phi(t))$. Define $V^a_{i,j}$ as the value of a floating-rate mortgage loan at the (i,j) node, and assume that the loan matures at time $t_0+m\Delta t$, i.e., $T=m\Delta t$ such that $V^a_{m,j}=\phi_j^2$ for each j. We use Figure 2 to demonstrate how we grid time period t and the state variable $\phi(t)$ to find $V^a_{i,j}$.

Insert Figure 2 here

The partial derivatives of $V^a(t,\phi(t))$, with respect to ϕ at node $(i-1,j)$, are approximated as follows:

$$\frac{\partial V^a(t,\phi(t))}{\partial \phi(t)} = \frac{V^a_{i,j+1} - V^a_{i,j-1}}{2\Delta\phi}, \tag{A1}$$

$$\frac{\partial^2 V^a(t,\phi(t))}{\partial \phi(t)^2} = \frac{V^a_{i,j+1} + V^a_{i,j-1} - 2V^a_{i,j}}{(\Delta\phi)^2}, \tag{A2}$$

and the time derivative is approximated as

$$\frac{\partial V^a(t,\phi(t))}{\partial t} = \frac{V^a_{i,j} - V^a_{i-1,j}}{\Delta t}. \tag{A3}$$

Because the short-term interest rate is $\phi(t)^2$, we can calculate the value of the loan as follows (see, Hull and White, 1990):

$$V^a_{i,j} = \frac{1}{1+\phi_j^2 \Delta t}[\frac{1}{6}V^a_{i+1,j-1} + \frac{2}{3}V^a_{i+1,j} + \frac{1}{6}V^a_{i+1,j+1} + \phi_j^2], \tag{A4}$$

for $i = 1, 2, \ldots, m-1$, and $j = 1, 2, \ldots, n-1$,

$$V^a_{i,0} = \frac{1}{1+\phi_0^2 \Delta t}[\frac{1}{6}V^a_{i+1,0} + \frac{2}{3}V^a_{i+1,1} + \frac{1}{6}V^a_{i+1,2} + \phi_j^2], \tag{A5}$$

and

$$V^a_{i,n} = \frac{1}{1+\phi_n^2 \Delta t}[\frac{1}{6}V^a_{i+1,n-2} + \frac{2}{3}V^a_{i+1,n-1} + \frac{1}{6}V^a_{i+1,n} + \phi_j^2], \tag{A6}$$

Now consider the value of the fixed-rate mortgage loan with prepayment options, $V^f(t, r(t), d)$. We will ignore the argument d and define $V^f_{i,j}$ as the value of the loan at the (i, j) node. We use the Figure 3 to show the motions of ϕ_j and $V^f_{i,j}$. When the interest rate decreases from ϕ_j at $t = i$ to ϕ_{j-1} at $t = i+1$, it is possible for the fixed-rate borrower to pay penalties in exchange for a floating-rate mortgage loan with a lower contract rate. Specifically, if $V^f_{i+1,j-1} > V^a_{i+1,j-1} + \alpha$, then the fixed-rate borrower will do so. Otherwise, the fixed-rate borrower will pay interest equal to $(1+d)\phi(t_0)^2$. When the interest rate increases or remains unchanged, then the fixed-rate borrower will also pay $(1+d)\phi(t_0)^2$.

Knowing the condition for prepayment, we can then calculate $V_{i,j}^{f}$, which resembles Equation (A4)-(A6), except that we need to change ϕ_j^2 by $(1+d)\phi(t_0)^2$ and impose $V_{i,j}^{f}$ $(j=0,...,n)$ by $V_{i,j}^{a}+\alpha$ if the fixed-rate borrower pays off the loan balance. We also require the equilibrium condition $V_{0,j}^{a}=V_{0,j}^{f}$ to solve the equilibrium value of d, d^*. This equilibrium condition and the condition of whether the fixed-rate loan borrower prepays the loan are jointly determined in the simulation analysis.

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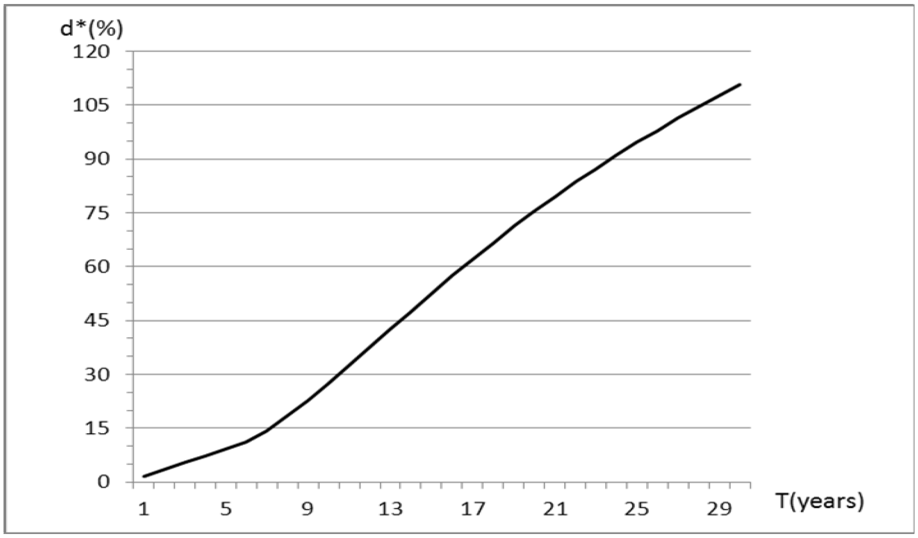


Figure 1: The interest rate differential, d^*, as a function of the term of the loan, T. This graph shows that when the term of the loan is lengthened from one year to thirty years, the interest rate differential increases.

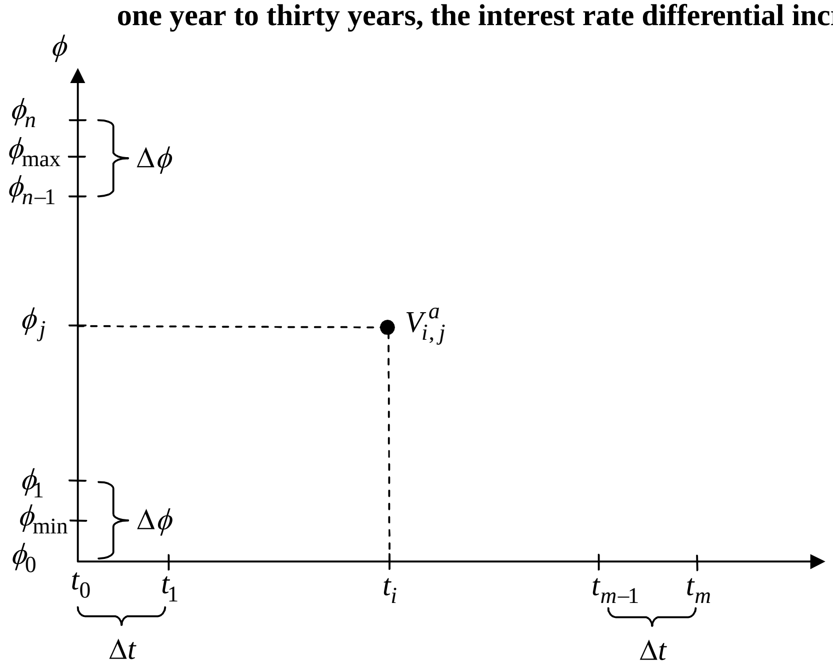
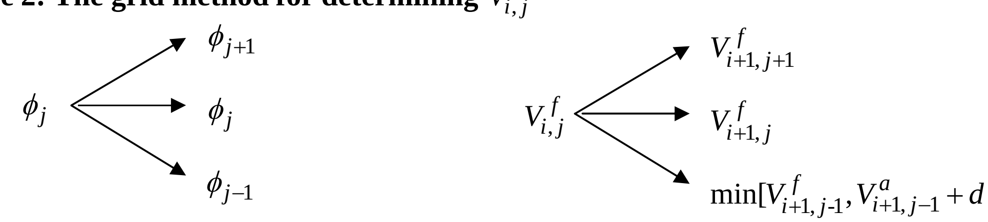


Figure 2: The grid method for determining $V_{i,j}^a$



(a) (b)

Figure 3: The motion of ϕ_j and $V_{i,j}^{f}$. When the interest rate increases or remains unchanged (from ϕ_j to ϕ_j or ϕ_{j+1}), the fixed-rate mortgage borrower will not act. If the interest rate decreases from ϕ_j or ϕ_{j-1}, then the borrower will (will not) prepay the loan if $V_{i+1,j-1}^{f} > (<)\ V_{i+1,j-1}^{a} + \alpha$.

Table 1: The interest rate differential in equilibrium, d^* (%)

a (%)	18	19	20	21	22
d^*(%)	9.34	9.21	9.07	8.95	8.82
b (%)	5	5.5	6	6.5	7
d^*(%)	9.54	9.30	9.07	8.86	8.65
c (%)	8	9	10	11	12
d^*(%)	5.50	7.05	9.07	11.9	12.6
$r(t_0)$(%)	4	5	6	7	8
d^*(%)	14.5	11.3	9.07	7.58	6.59
α (%)	15	17.5	20	22.5	25
d^*(%)	9.73	9.25	9.07	9.04	9.03

Note: The benchmark parameter values are as follows: T = 5 years; $a = 20\%$ per year; $b = 6\%$ per year; $c = 10\%$ per year; $r(t_0) = 6\%$ per year; and $\alpha = 20\%$. Given this set of benchmark parameter values, in equilibrium, the interest rate differential, d^*, is equal to 9.07%, meaning that the fixed-rate loan borrower will be charged at 6.54% per year at the initial date. Additionally, T, a, b, $c/\sqrt{r(t_0)}$, $r(t_0)$, and α denote the term of the loan, the reversion speed, the long-run steady state of interest rates, the volatility of the growth rate of interest rates, the current market rate of interest, and the transaction cost parameter, respectively.